Structural Empirical Work Using Matching Models

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Economists often observe data on relationships. We see who is in a relationship with whom: which firms merged with each other, which men are married to which women, and which bidders won which items in an auction. A matching model or matching game is one theoretical framework for modeling the equilibrium formation of these relationships. A relationship is termed a match. A matching model takes a set of payoffs or outputs for all possible matches and produces a set of matches where no couple would prefer to deviate and become matched, instead of their assigned matches. The robustness of the equilibrium to deviation by any potential couple suggests "pairwise stability" as the term for an equilibrium.

The key economic idea in matching models is the rivalry to match with the most attractive partners. In marriage, men compete with each other other to marry the most attractive women while women compete with each other to marry the most attractive men. There is scarcity on both sides of the market.

This entry focuses on research that formally estimates the parameters of a matching model using data on who matches with whom. Economists adopting this structural approach typically observe data on who matches with whom as well as exogenous characteristics of each agent. For example, economists studying marriage will observe the race, schooling level, religion, physical attractiveness and wage of each man and each women. The data also record which men married which women. Economists are willing to assume that the data represent a pairwise stable outcome to a particular matching game.

The structural approach means that researchers impose the structural model and estimate unknown parameters in the model. The advantages of the structural approach apply to more economic situations than just matching models and have been explored elsewhere (Reiss and Wolak, 2007). A quick summary would be that the structural approach allows the computation of counterfactuals and the measurement of economic parameters that cannot be directly observed. Using the example of marriage, one type of counterfactual would be to explore how the equilibrium set of matches is altered as demographics change. Measurement is also important: if men and women each have several characteristics, how important are each of the characteristics in the payoffs to a match?

This paper focuses on the use of matching games in structural empirical work. Other literatures have focused on centralized market design (for example the medical resident matching program in the United States) and the descriptive interpretation of matching patterns.

1 Two-Sided Matching Games

Most but not all empirical work has focused on two-sided matching games: agents can be divided into two sides, say men and women. Roth and Sotomayor (1990) is a useful text that explains simple matching games.

For the purposes of this article, I will divide two-sided matching games into models with and without transfers. Gale and Shapley (1962) introduced the model where agents do not exchange money: men have preferences over women and women have preferences over men. Generically there will be a lattice of multiple pairwise stable outcomes in this model. Koopmans and Beckmann (1957), Shapley and Shubik (1972), and Becker (1973) study models where matched agents can exchange money and where agents have transferable utility. For one-to-one matching games such as marriage, generically there will be one set of pairwise stable physical matches in these models; there may be a continuum of transfers that support these physical matches. The choice of modeling framework depends on the researcher's understanding of the market in question.

The above papers allow each man, say, to marry at most one woman. There are extensions to many-to-one and many-to-many two-sided matching games. Complementarities between multiple matches involving the same agent are key to some of the empirical applications below (Fox, 2009a; Fox and Bajari, 2009). There are also one- and many-sided matching games.

There are more general matching games where other contract elements, such as the hours of work in a labor market, are determined as part of the pairwise stable outcome (Crawford and Knoer, 1981; Kelso and Crawford, 1982; Hatfield and Milgrom, 2005). Matching games are mathematically linked to hedonic equilibrium models, although I will not explore the link here (Rosen, 1974). There is also a clear link to models of frictions, such as search models, that also have observed agent heterogeneity (Shimer and Smith, 2000; Atakan, 2006).

2 Estimation methods

Matching games share many similarities with the literature on estimating static, discrete Nash games, such as the well-known entry models of Berry (1992) and Bresnahan and Reiss (1991).

Matching games use pairwise stability and not Nash equilibrium, but many estimation challenges are similar. A key difficulty in matching games is that the number of agents in a market can be in the hundreds or thousands, compared to the three or four firms deciding to enter a market in some entry applications. The number of agents in matching empirical applications can make some estimators computationally infeasible.

2.1 Nested solution methods

The most straightforward way to estimate a matching game is to use simulated maximum likelihood or the simulated method of moments. These estimators require solving the model a fixed number of times for each iteration of an outer optimization routine. Simulation estimators are conceptually straightforward but computationally burdensome.

Boyd, Lankford, Loeb and Wyckoff (2003) use the simulated method of moments to study the matching of public school teachers to schools in New York state. They use data on wages and assume the wages are exogenously determined. Their model without endogenous transfers has multiple stable matches and they use the lattice structure of the equilibria to impose an equilibrium selection rule in estimation.

2.2 Full likelihood methods

In many cases, the full likelihood can be written down. In a study of the matching of venture capitalists to entrepreneurs, Sørensen (2007) uses an augmented likelihood approach where the unobserved payoffs of each match are treated as nuisance parameters and integrated out using a blocking structure in a Markov Chain Monte Carlo (MCMC) procedure. Sørensen does not use endogenous transfers and hence imposes an aligned preferences assumption that he proves generates a unique pairwise stable outcome.

The full likelihood approach can be computationally intractable in large matching markets. In a study of the matching between investment banks and firms undertaking an initial public offering, Akkus (2008) shows that the likelihood simplifies if the values of realized matches are recorded in the data. By using data on the payoffs of matches, estimation becomes easier.

2.3 Inequality methods

With an application to automotive suppliers and automotive assemblers, Fox (2009a) introduces a maximum score estimator to estimate a many-to-many matching game where transfers are endogenous, but not in the data. The maximum score estimator maximizes the number of inequalities implied by pairwise stability that hold true. This approach breaks the computational curse of dimensionality because not all inequalities need to be included for the estimator to be consistent.

2.4 Logit methods

Dagsvik (2000) and Choo and Siow (2006) study games with transfers and assume that the payoffs to matches have error terms that satisfy the parametric logit property. To a large degree, the logit assumption allows researchers to derive closed-form equations that allow estimation, especially for very large markets that plausibly have a continuum of agents, such as the US national marriage market.

2.5 Identification

In matching games, agents on the same side of the market are rivals to match with agents on the other side of the market. The fact that a man did not match with the most attractive woman does not mean that the man did not prefer that woman to his actual wife. The equilibrium budget set of each agent is unobserved. Thus, it is not clear what can be learned (identified) from data on who matches with whom.

Fox (2009b) studies identification in matching games with transfers and finds two sets of results. First, the relative importance of complementarities in payoffs for, say, schooling compared to, say, wealth is identified using data on matches but not the equilibrium transfers that are present in the model. Second, the ordering of production levels (which matches give higher payoffs) is identified using the same data.

2.6 Selection correcting outcome equations

Sørensen (2007) explores the use of a matching model to parametrically selection correct an auxiliary outcome equation. The outcome, the success of an investment in his application, is not determined as part of the matching game, but the outcome is only observed in the data for realized matches. Sørensen's approach is analogous to using a single agent decision model to selection correct an outcome equation (Heckman, 1979). Sørensen (2009) extends the framework of Heckman (1990) to study identification in selection models where selection is induced by a matching game.

3 Empirical applications

I now catalog some of the many empirical applications of matching games.

3.1 Marriage and family economics

Choo and Siow (2006) explore whether changing matching patterns in the US marriage market are due to changes in preferences or changes in the exogenous characteristics of potential spouses. They also explore the effects of the legalization of abortion. There have been a large number of marriage and family economics papers following up on the Choo and Siow framework, many by the original authors. See Siow (2008) for a complete survey of this material.

Bruze (2009) estimates a matching game where labor supply and the split of consumption between men and women are part of the pairwise stable contract terms. He explores the return to an agent for finding a higher-earning spouse in college.

Hitsch, Hortaçsu and Ariely (2009) use revealed preference information from an online dating site to avoid the need to use an equilibrium model to estimate preferences. They use the preference estimates to simulate a pairwise stable outcome and find it matches actual sorting on the site well.

3.2 Industrial organization, corporate finance and marketing

Hall (1988) is an early paper that emphasizes the need for matching models to study mergers. She did not estimate a full matching game because of computational concerns.

It is common to have data on realized interfirm relationships. Sørensen (2007) studies the matching of venture capitalists to entrepreneurs with a focus on selection correcting an outcome equation where the success of an investment is regressed upon the experience of the venture capitalist. The basic approach of Sørensen allows for match-specific error terms, and he can allow for time-invariant characteristics of a venture capitalist by using fixed effects / panel data. Chen (2009) uses a similar selection-correction framework where the outcome equation of interest is the price of a bank loan. Akkus (2008) uses the selection-correction approach to regress the degree of first-day underpricing on the experience of an investment bank. Park (2008) uses a similar MCMC estimator to investigate the decision of a mutual fund manager to engage in a merger as a function of past returns.

Fox and Bajari (2009) was the first paper to estimate a many-to-one matching game where complementarities across multiple matches were allowed for. The authors look at auctions of multiple heterogeneous items, where each bidder can win multiple items. They study FCC spectrum auctions, where complementarities between the geographic territories being auctioned are estimated to be important for the efficient operation of the mobile phone industry. A key methodological challenge is showing how a potentially inefficient, dynamic Nash game could result in equilibria that satisfy pairwise stability. The estimator being used is that of Fox (2009a) for matching games with transfers. Fox (2009a) studies the many-to-many matching of automotive suppliers to automotive assemblers and measures the relative importance of specialization by suppliers in particular corporations, brands, and car models. Further, Fox measures a potential benefit of suppliers matching with high-quality Asian assemblers, such as Toyota.

Levine (2008) uses the estimator of Fox (2009a) to explore the matching of biotechnology innovations to marketing firms. She explores whether the returns to scale of marketers might decrease the returns to innovators. Yang, Shi and Goldfarb (2009) use the same estimator to explore the matching of professional athletes to teams, with a focus on the potential marketing complementarities between players and teams from different sized cities. Akkus and Hortacsu (2007) extend the maximum score estimator to use data on equilibrium transfers. Akkus and Hortacsu investigate geographic complementarities in the market for bank mergers after the removal of prohibitions against interstate banks. Mindruta (2009) studies the matching of university researchers and private firms.

3.3 Public finance, labor economics and other

Boyd, Lankford, Loeb and Wyckoff (2003) investigate the matching of teachers to public schools, with a focus on learning how to attract qualified teachers to schools in impoverished areas.

Gordon and Knight (2009) investigate the consolidation decisions of Iowa school districts after the state passed incentives inducing such consolidation.

Fox (2008) estimates a repeated matching model for the labor market for engineers in Sweden. Each period state variables evolve, a matching model opens, prices are formed to equate supply and demand and workers choose jobs. The model is dynamic in that both firms and workers are forward looking: they consider the effect of the decision to switch today on future outcomes.

Baccara, Imrohoroglu, Wilson and Yariv (2009) study the matching of professors to offices and estimate the importance of various types of professional networks in payoffs.

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